

Finite-size scaling in disordered systems

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(Received 11 July 2001; published 22 January 2002)

The critical behavior of a quenched random hypercubic sample of linear size L is considered, within the “random- T_c ” field-theoretical model, by using the renormalization group method. A finite-size scaling behavior is established and analyzed near the upper critical dimension $d=4-\epsilon$ and some universal results are obtained. The problem of self-averaging is clarified for different critical regimes.

DOI: 10.1103/PhysRevE.65.026129

PACS number(s): 75.10.Nr, 05.70.Fh, 64.60.Ak, 75.40.Mg

I. INTRODUCTION

The description of effects of disorder on the critical behavior of finite-size systems has attracted a lot of interest [1–8]. Up to now, discussions have taken place on whether the introduced disorder influences the finite-size scaling (FSS) results [3,8], compared to the standard FSS results, known for pure systems [9–11]. A formulation of general FSS concepts for the case of disorder is strongly complicated due to the additional averaging over the different random samples. For a random sample with volume L^d , where L is a linear dimension, any observable property X , singular in the thermodynamic limit, has different values for different realizations of the randomness and can be considered as a stochastic variable with mean \bar{X} and variance $(\Delta X)^2 := \overline{X^2} - \bar{X}^2$, where the overline indicates an average over all realizations of the randomness. Here, an important theoretical problem of interest is related with the property of self-averaging (SA) [12]. If the system does not exhibit SA a measurement performed on a single sample does not give a meaningful result and must be repeated on many samples. A numerical study of such a system also will be quite difficult. This point has been studied recently by means of FSS arguments [1,4], renormalization group (RG) analysis [2,5] and Monte Carlo simulations [4,6]. The quantity under inspection is the relative variance $R_X(L) := (\Delta X)^2 / \bar{X}^2$. A system is said to exhibit “strong SA” if $R_X(L) \sim L^{-d}$ as $L \rightarrow \infty$. This is the case if the system is away from criticality, i.e., if $L \gg \xi$.

At criticality, i.e., when $L \ll \xi$, the situation depends on whether the randomness is irrelevant or relevant. In the former case the relative variance $R_X(L) \sim L^{\alpha_{\text{pure}}/\nu_{\text{pure}}}$, where α_{pure} and ν_{pure} are the specific heat and correlation length exponents respectively, for the corresponding “pure” system. Since their ratio is small and nonpositive, this leads to “weak SA” [2].

In the latter case, when randomness is relevant, the correlation length exponent of the random system $\nu_{\text{random}} > 2/d$ [2], and the system exhibits “no SA,” as $R_X(L)$ does not

vanish and is a fixed nonzero universal quantity even in the thermodynamic limit.

The lack of SA in disordered systems implies that the standard FSS breaks down. Thus, one needs to formulate a FSS theory suitable for the case under consideration. There is an ongoing activity in this field but an understanding of the problems at a deeper level is still desirable [1,3,8].

The successful application of the field-theoretical methods [13,14] for the analytic calculations of the size-dependent universal scaling functions for pure systems makes the possible extension of these methods very appealing also in the case of disordered systems. The appropriate field theory is the N -component ψ^4 theory with a “random- T_c ” term [15–21]. The analytic difficulties raised by the disorder usually are avoided with the replica trick by solving an effective pure problem [17]. However, the perturbative structure of the theory is still much more complicated than for the corresponding pure system, raising additional difficulties in the applicability of the ideas proposed in [13,14]. Some of them are of rather hard computational nature.

Between the generic models for magnetism, the most popular and relatively well-studied model by means of Monte Carlo simulations is the Ising model (i.e., $N=1$) due to the fact that in three dimensions this is the model for which, in accordance with the Harris criterion, randomness is relevant. But its RG analysis is complicated as a result of the well-known accidental degeneracy in the recursion relations that needs higher order in ϵ and so the use of the loop expansion to second order (see, e.g., [15,18–20]). This leads to the apparent computational difficulties in the finite-size treatment of the system. The ϵ calculations based on the loop expansion to second order are not done even for the pure finite-size systems. In this situation, more attractive for the finite-size RG study are the disordered XY ($N=2$) and Heisenberg ($N=3$) cases, as these are simpler and have the same qualitative features as governed in $d=4-\epsilon$ dimensions by a random fixed point.

Other problems have a more basic nature and are related with the breaking of the replica symmetry (see [22]). Since its deeper understanding is still lacking, they are beyond our interest in the present study.

In this paper we analyze the finite-size properties of an N -component ($N>1$) model of a randomly diluted magnet with hypercubic geometry of linear size L . Exact calculations

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are performed in the mean-field regime $d > 4$, and up to the first order in ϵ near the upper critical dimension $d_c = 4$. Although in this case, the problem of usefulness of the corresponding series expansions away from the dimension $d = 4 - \epsilon$ arises and is questionable (see [23] and references therein), we shall show that many generic FSS properties of the model can be established and we hope this would have implications for more realistic cases.

The paper is organized as follow. In Secs. II and III we define the model and the effective Hamiltonian. In Sec. IV we perform the analysis in the zero-mode approximation. In Secs. V and VI we give the expressions for the shift of the critical temperature and the renormalized coupling constants in first order in ϵ . Section VII deals with the verification of the FSS, and the analysis of the problem of SA is given in Sec. VIII. Finally in Sec. IX we present our main conclusions.

II. MODEL

We consider the “random- T_c ” Ginzburg-Landau-Wilson model of disordered ferromagnets (see, e.g., [15–20]),

$$\mathcal{H}_r = -\frac{1}{2} \int_{L^d} d^d x \left[t |\psi(\mathbf{x})|^2 + \varphi(\mathbf{x}) |\psi(\mathbf{x})|^2 + c |\nabla \psi(\mathbf{x})|^2 + \frac{u}{12} |\psi(\mathbf{x})|^4 \right], \quad (2.1)$$

where $\psi(\mathbf{x})$ is an N -component field with $\psi^2(\mathbf{x}) = \sum_{i=1}^N \psi_i^2(\mathbf{x})$, and the random variable $\varphi(\mathbf{x})$ has a Gaussian distribution

$$P(\varphi(\mathbf{x})) = \frac{\exp\left[-\frac{\varphi(\mathbf{x})^2}{2\Delta}\right]}{\sqrt{2\pi\Delta}} \quad (2.2)$$

with mean

$$\overline{\varphi(\mathbf{x})} = 0 \quad (2.3)$$

and variance

$$\overline{\varphi(\mathbf{x})\varphi(\mathbf{x}')} = \Delta \delta^d(\mathbf{x} - \mathbf{x}'). \quad (2.4)$$

The overline in Eq. (2.4) indicates a random average performed with the distribution $P(\varphi(\mathbf{x}))$. Here we will consider a system in a finite cube of volume L^d with periodic boundary conditions. This means that the following expansion takes place:

$$\psi(\mathbf{x}) = \frac{1}{L^d} \sum_{\mathbf{k}} \tilde{\psi}(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{x}) \quad (2.5)$$

and

$$\varphi(\mathbf{x}) = \frac{1}{L^d} \sum_{\mathbf{k}} \tilde{\varphi}(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{x}), \quad (2.6)$$

where \mathbf{k} is a discrete vector with components $k_i = 2\pi n_i/L$, $n_i = 0, \pm 1, \pm 2, \dots$, $i = 1, \dots, d$ and a cutoff $\Lambda \sim a^{-1}$ (a is the lattice spacing). In this paper, we are interested in the continuum limit, i.e., $a \rightarrow 0$.

In our case of quenched randomness one must average the logarithm of the partition function over the Gaussian distribution (2.2) to produce the free energy

$$\mathcal{F}[\mathcal{H}_r] = - \int_{-\infty}^{\infty} D\varphi(\mathbf{x}) P(\varphi(\mathbf{x})) \ln \mathcal{Z}_r; \quad (2.7)$$

where

$$\mathcal{Z}_r = \text{Tr}_{\psi} \exp[\mathcal{H}_r]. \quad (2.8)$$

It is well known that the direct average over the Gaussian leads to equivalent results [24] for the critical behavior in the $n=0$ limit of the following “pure” translationally invariant model [25]:

$$\begin{aligned} \mathcal{H}_p(n) = & -\frac{1}{2} \sum_{\alpha=1}^n \int_{L^d} d^d x \left[t |\psi_{\alpha}(\mathbf{x})|^2 + c |\nabla \psi_{\alpha}(\mathbf{x})|^2 \right. \\ & \left. + \frac{u}{12} |\psi_{\alpha}(\mathbf{x})|^4 \right] \\ & + \frac{\Delta}{8} \sum_{\alpha,\beta=1}^n \int_{L^d} d^d x |\psi_{\alpha}(\mathbf{x})|^2 |\psi_{\beta}(\mathbf{x})|^2. \end{aligned} \quad (2.9)$$

Here $\psi_{\alpha}(\mathbf{x})$, $\alpha = 1, \dots, n$ (n being the number of replicas) are components of an $(n \times N)$ -component field $\bar{\psi}(\mathbf{x})$. Owing to this equivalence, the model \mathcal{H}_p has been the object of intensive field-theoretical studies (see [23] and references therein) in the bulk case. Much less is known for the equivalence of \mathcal{H}_r and the $n=0$ limit of \mathcal{H}_p in the finite-size case. Problems may arise when finite-size techniques are used, since both the procedures $L \rightarrow \infty$ and removing of the disorder by the “trick” $n \rightarrow 0$ may not commute.

III. THE EFFECTIVE HAMILTONIAN

In this work we will use the RG technique introduced in [13,14] for studying pure systems with finite geometry. This technique permits explicit analytical calculations above and in the neighborhood of the upper critical dimension. The main idea is to expand the field in Eq. (2.1) in Fourier modes and then to treat the zero mode separately from the nonzero modes. The nonzero modes can be treated by the methods developed for the bulk systems (e.g., loop expansion), while the zero mode, whose fluctuations are damped at the critical temperature, has to be treated exactly. This overcomes the problems due to the infrared (IR) divergences that take place in finite-size systems (see Ref. [26]).

In our more complicated case we have two possibilities: to consider the random model equation (2.1) or to consider the replicated pure model equation (2.9). The last one is closer to the case treated in [13,14] by getting around the difficulties due to the random average performed with $P(\varphi(\mathbf{x}))$ and is used in the present study. For this case, the

replicated partition function is given by

$$\mathcal{Z}_p(n) = \int \mathcal{D}\vec{\psi} \exp[\mathcal{H}_p(n)]. \quad (3.1)$$

We decompose the field $\vec{\psi}(\mathbf{x})$ into a zero-momentum component $\vec{\phi} = L^{-d} \int d^d x \vec{\psi}(\mathbf{x})$, which plays the role of the uniform magnetization, and a second part depending upon the nonzero modes $\vec{\sigma} = L^{-d} \sum_{\mathbf{k} \neq 0} \vec{\psi}(\mathbf{k}) \exp(-i\mathbf{k} \cdot \mathbf{x})$. After some algebra, the partition function can be expressed as

$$\begin{aligned} \mathcal{Z}_p(n) = \int \mathcal{D}\phi \mathcal{D}\sigma \exp \left\{ -\frac{L^d}{2} \sum_{\alpha=1}^n \left(r_0 \phi_\alpha^2 + \frac{u_0}{12} \phi_\alpha^4 \right) \right. \\ \left. + \frac{L^d \Delta_0}{8} \left(\sum_{\alpha=1}^n \phi_\alpha^2 \right)^2 - \frac{1}{2} \sum_{\alpha=1}^n \sum_{\mathbf{k}} \left[r_0 + \mathbf{k}^2 + \frac{u_0}{2} \phi_\alpha^2 \right. \right. \\ \left. \left. - \frac{\Delta_0}{2} \sum_{\beta=1}^n \phi_\beta^2 \right] \sigma_\alpha^2 + (\text{higher powers of } \sigma) \right\}. \end{aligned} \quad (3.2)$$

Here the terms involving $\int d^d x \sigma$ vanish since σ depends only on nonzero modes. The terms containing σ are treated using diagram expansion, leading to the effective Hamiltonian in the one-loop approximation,

$$\begin{aligned} \mathcal{H}_p^{\text{eff}}(u) = -\frac{L^d}{2} \sum_{\alpha=1}^n \left(\tilde{t}(n) \phi_\alpha^2 + \frac{\tilde{u}(n)}{12} \phi_\alpha^4 \right) \\ + \frac{L^d \tilde{\Delta}(n)}{8} \left(\sum_{\alpha=1}^n \phi_\alpha^2 \right)^2, \end{aligned} \quad (3.3)$$

where $\tilde{t}(n)$, $\tilde{u}(n)$, and $\tilde{\Delta}(n)$ will be presented below. With the help of the identity

$$\exp\left(\frac{aA^2}{2}\right) = \frac{1}{(2\pi a)^{1/2}} \int_{-\infty}^{\infty} dy \exp[-(1/2a)y^2 + yA], \quad (3.4)$$

we get

$$\begin{aligned} \mathcal{Z}_p^{\text{eff}}(n) = \text{Tr}_\phi \exp[\mathcal{H}_p^{\text{eff}}(n)] = \int_{-\infty}^{\infty} dy P_n(y) \\ \times \left[S_N \int_0^\infty d|\phi| |\phi|^{N-1} \exp\{\mathcal{H}_r^{\text{eff}}(n)\} \right]^n, \end{aligned} \quad (3.5)$$

where

$$\mathcal{H}^{\text{eff}}(n) = -\frac{1}{2} L^d \left[\left(\tilde{t}(n) + \frac{y}{L^{d/2}} \right) |\phi|^2 + \frac{1}{12} \tilde{u}(n) |\phi|^4 \right] \quad (3.6)$$

is an effective Hamiltonian with a random variable y with Gaussian distribution [depending on $\tilde{\Delta}(n)$]

$$P_n(y) = \frac{\exp\left(-\frac{y^2}{2\tilde{\Delta}(n)}\right)}{\sqrt{2\pi\tilde{\Delta}(n)}} \quad (3.7)$$

and $S_N = 2\pi^{N/2}/\Gamma(N/2)$ is the surface of an N -dimensional unit sphere.

Let us note that the above mentioned equivalence between the models (2.1) and (2.9) may be mathematically expressed, within the used approximation, by the following relation:

$$\mathcal{F}[\mathcal{H}_r] = -\frac{\partial}{\partial n} \mathcal{Z}_p(n) \Big|_{n=0}. \quad (3.8)$$

From Eqs. (3.5) and (3.8), and by using the identity

$$\frac{\partial}{\partial n} A^n(n) \Big|_{n=0} = \ln A(0) \quad (3.9)$$

for the free energy, we get

$$\mathcal{F}[\mathcal{H}_r] = -\int_{-\infty}^{\infty} dy P_0(y) \ln \mathcal{Z}_r(0), \quad (3.10)$$

where

$$\mathcal{Z}_r(0) = S_N \int_0^\infty d|\phi| |\phi|^{N-1} \exp[\mathcal{H}_r^{\text{eff}}(0)] \quad (3.11)$$

is the partition function for the random system (3.6) after taking the limit $n \rightarrow 0$. The obtained effective ‘‘random- T_c model’’ (3.6), distributed with Gaussian weight (3.7), is the analytic basis of this paper. The effective constants $\tilde{t}(n)$, $\tilde{u}(n)$, and $\tilde{\Delta}(n)$ involve n and finite-size L as parameters. For describing the finite-size properties of the initial model (2.1), as follows from Eqs. (3.10) and (3.11), it is necessary to set n to zero. In the following sections we shall consider the results of this procedure.

IV. THE FSS EXPRESSION FOR THE FREE ENERGY AND CUMULANTS IN THE ZERO-MODE APPROXIMATION

If we neglect the loop corrections this corresponds to the mean-field (MF) approximation, i.e., $d > 4$. Then the zero mode playing the role of the uniform magnetization may be treated exactly. In this case the effective Hamiltonian (3.3) of the model reduces to

$$\mathcal{H}_p^{\text{MF}} = -\frac{1}{2} L^d \left[t \sum_{\alpha=1}^n \phi_\alpha^2 + \frac{1}{12} u \sum_{\alpha=1}^n (\phi_\alpha^2)^2 - \frac{\Delta}{4} \left(\sum_{\alpha=1}^n \phi_\alpha^2 \right)^2 \right]. \quad (4.1)$$

Now using an appropriate rescaling of the field $|\phi| = (uL^d)^{-1/4} \Phi$ and introducing the scaling variable

$$\mu = tL^{d/2} u^{-1/2} \quad (4.2)$$

for the partition function (3.11) in the mean-field approximation, we obtain

$$\mathcal{Z}_r^{\text{MF}}(0) = (uL^d)^{-N/4} \mathcal{I}_N(\mu + y/u^{1/2}), \quad (4.3)$$

where we have introduced the following auxiliary function:

$$\mathcal{I}_N(z) = S_N \int_0^\infty d\Phi \Phi^{(N-1)} \exp\left\{-\frac{1}{2}[z\Phi^2 + \frac{1}{12}\Phi^4]\right\}. \quad (4.4)$$

From Eqs. (3.10) and (4.3) we get for the free energy

$$\begin{aligned} \mathcal{F}[\mathcal{H}_r^{\text{MF}}] = & -\frac{1}{\sqrt{2\pi\Delta}} \int_{-\infty}^\infty dy \exp\left(-\frac{1}{2}\frac{y^2}{\Delta}\right) \ln[(uL^d)^{-N/4} \\ & \times \mathcal{I}_N(\mu + y/u^{1/2})]. \end{aligned} \quad (4.5)$$

If we introduce a second scaling variable

$$\lambda = \frac{\Delta}{u}, \quad (4.6)$$

and use Eqs. (A4) (see the Appendix), Eq. (4.5) takes its final form,

$$\begin{aligned} \mathcal{F}[\mathcal{H}_r^{\text{MF}}] = & -\frac{1}{\sqrt{2\pi\lambda}} \int_{-\infty}^\infty dx \exp[-(x-\mu)^2/(2\lambda)] \\ & \times \ln[D_{-N/2}(\sqrt{3}x)] - \frac{3}{4}(\lambda + \mu^2) \\ & + \frac{N}{4} \ln\left(\frac{uL^d}{12\pi^2}\right). \end{aligned} \quad (4.7)$$

For the Ising case $N=1$ a similar expression for the quenched free energy in a slightly different context is obtained and its analytic structure is studied in Refs. [27–29]. Obviously Eq. (4.7) is well defined for any positive λ and in the limit $\lambda \rightarrow 0$ we recover the well-known result for the free energy of the pure model.

In addition to the free energy, one also needs to know the correlation functions. Within the replica method the averages of the fields $\{\phi_\beta\}$ are defined by (see, e.g., [12])

$$\begin{aligned} \overline{\langle |\phi_\beta|^{2m} \rangle}_{\mathcal{H}_r^{\text{MF}}} = & \lim_{n \rightarrow 0} \left[\mathcal{Z}_p^{\text{MF}}(n)^{-1} S_N^n \int \left(\prod_{\alpha=1}^n d|\phi_\alpha| \right) \right. \\ & \left. \times (|\phi_\alpha|)^{N-1} (|\phi_\beta|)^{2m} \exp(\mathcal{H}_p^{\text{MF}}) \right], \end{aligned} \quad (4.8)$$

where

$$\mathcal{Z}_p^{\text{MF}}(n) = S_N^n \int \left(\prod_{\alpha=1}^n d|\phi_\alpha| \right) (|\phi_\alpha|)^{N-1} \exp(\mathcal{H}_p^{\text{MF}}). \quad (4.9)$$

Note that the final result must be independent of replica index β , because $\mathcal{H}_p^{\text{MF}}$ is invariant under permutation of the replicas. After taking the limit $n \rightarrow 0$, we end up with the following expression:

$$\begin{aligned} \overline{\mathcal{M}_{2m}} & := \overline{\langle |\phi_\beta|^{2m} \rangle}_{\mathcal{H}_r^{\text{MF}}} \\ & = \frac{(uL^d)^{-m/2}}{\sqrt{2\pi\lambda}} \int_{-\infty}^\infty dx \frac{\mathcal{I}_{N+2m}(x)}{\mathcal{I}_N(x)} e^{-(x-\mu)^2/2\lambda}. \end{aligned} \quad (4.10)$$

In a similar way

$$\begin{aligned} \overline{(\mathcal{M}_2)^2} & := \overline{\langle |\phi_\alpha|^2 |\phi_\beta|^2 \rangle}_{\mathcal{H}_r^{\text{MF}}} \\ & = \frac{(uL^d)^{-1}}{\sqrt{2\pi\lambda}} \int_{-\infty}^\infty dx \left[\frac{\mathcal{I}_{N+2}(x)}{\mathcal{I}_N(x)} \right]^2 e^{-(x-\mu)^2/2\lambda}. \end{aligned} \quad (4.11)$$

Equations (4.10) and (4.11) show that the finite-size scaling depends on two scaling variables μ from Eq. (4.2) and λ from Eq. (4.6). Only the first one is size dependent, having the same form as in the pure case [13]. This result leads to the prediction that above four dimensions, the weak disorder does not affect the finite-size scaling proven for pure systems. From Eqs. (4.10) and (4.11), when $\mu=0$ and $N=1$, we obtain the results of Ref. [6].

In terms of the momenta $\overline{\mathcal{M}_{2p}}$ the susceptibility is given as

$$\chi = L^d \overline{\mathcal{M}_2}. \quad (4.12)$$

Another quantity of importance for numerical analysis is the Binder cumulant defined by

$$B = 1 - \frac{1}{3} \frac{\overline{\mathcal{M}_4}}{\overline{\mathcal{M}_2}^2} \quad (4.13)$$

and the cumulant specific for the random system defined as

$$R = \frac{(\overline{\mathcal{M}_2})^2 - \overline{\mathcal{M}_2}^2}{\overline{\mathcal{M}_2}^2}. \quad (4.14)$$

Since the parameter R is the relative variance of the observable (the susceptibility), as we said in the Introduction, it is a measure of the SA in the random system. If SA takes place this quantity should be zero in the thermodynamic limit.

V. FINITE-SIZE SHIFT OF T_c : LOWEST ORDER IN ϵ

For $d < 4$, the perturbatively calculated parts of the free energy and cumulants, which contain contributions of all nonzero modes, depend, to one-loop order, on the shift of the critical temperature and on the renormalized coupling constants u and Δ . The application of the finite-size ϵ expansion

to the model system (2.9) requires the corresponding renormalization constants.

To one-loop order, using the minimal subtraction scheme, before taking the $n \rightarrow 0$ limit, we obtain

$$Z_t = 1 + \frac{N+2}{6\epsilon} \hat{u} - \frac{2+nN}{2\epsilon} \hat{\Delta}, \quad (5.1a)$$

$$Z_u = 1 + \frac{N+8}{6\epsilon} \hat{u} - \frac{6\hat{\Delta}}{\epsilon}, \quad (5.1b)$$

$$Z_\Delta = 1 + \frac{N+2}{3\epsilon} \hat{u} - \frac{8+nN}{2\epsilon} \hat{\Delta}. \quad (5.1c)$$

In Eqs. (5.1),

$$\hat{u} = u \frac{2}{(4\pi)^{d/2} \Gamma(d/2)}, \quad (5.2a)$$

$$\hat{\Delta} = \Delta \frac{2}{(4\pi)^{d/2} \Gamma(d/2)}. \quad (5.2b)$$

The β functions associated with \hat{u} and $\hat{\Delta}$ have the form

$$\beta_u = -\hat{u}\epsilon + \frac{N+8}{6} \hat{u}^2 - 6\hat{u}\hat{\Delta}, \quad (5.3a)$$

$$\beta_\Delta = -\hat{\Delta}\epsilon - \frac{8+nN}{2} \hat{\Delta}^2 + \frac{N+2}{3} \hat{u}\hat{\Delta}. \quad (5.3b)$$

The fixed points of this system first have been studied in [30] and for the purposes of the impurity problem in [17]. The values of \hat{u} and $\hat{\Delta}$ in the fixed point, interesting in the random case, are

$$\hat{u}^*(n) = \frac{6(4-nN)}{16(N-1)-nN(N+8)} \epsilon, \quad (5.4a)$$

$$\hat{\Delta}^*(n) = \frac{2(4-N)}{16(N-1)-nN(N+8)} \epsilon. \quad (5.4b)$$

The corresponding expression for the exponent ν up to the first order of ϵ is

$$\frac{1}{\nu(n)} = 2 - \frac{6N(1-n)}{16(N-1)-nN(N+8)} \epsilon. \quad (5.5)$$

It should be noted here that we shall consider N -component fields with $1 < N < N_c(d)$, where $N_c(d) = 4 - 4\epsilon + O(\epsilon^2)$ is the critical number of spin components that defines the stability of the random fixed point in the $n=0$ limit. The stability of the different fixed points of the model has been also considered in [31]. The analysis of the Ising case ($N=1$) needs to perform a loop expansion to second order (see Introduction) and is beyond the scope of the present study.

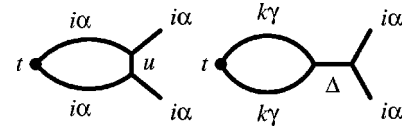


FIG. 1. One-loop contributions to the reduced temperature t .

As was explained above, the loop corrections will be treated perturbatively in the nonzero \mathbf{k} modes. In the lowest order in ϵ , this procedure generates a shift of the critical temperature $t \rightarrow \tilde{t}(n)$,

$$\tilde{t}(n) = tZ_t + t_L, \quad (5.6)$$

where the term tZ_t is coming from the one-loop counterterm [see Eq. (5.1)], and

$$t_L = \left[\frac{N+2}{6} \hat{u} - \frac{2+Nn}{2} \hat{\Delta} \right] \frac{1}{L^d} \sum'_{\mathbf{k}} \frac{1}{\mathbf{k}^2 + t} \quad (5.7)$$

is the finite-size correction. The two diagrammatic contributions for t_L are shown in Fig. 1. Both diagrams from the u and the Δ contributions differ only by their numerical factors. The prime in the d -fold sum in the above equations denotes that the term with a zero summation index has been omitted.

After some algebra [for details of the pure case ($\Delta=0$), see Ref. [13]], near the upper critical dimension $d=4-\epsilon$, we obtain

$$\tilde{t}(n) = t + \left[\frac{N+2}{12} \hat{u} - \frac{2+Nn}{4} \hat{\Delta} \right] [t \ln t + 4L^{-2} F_{d,2}(tL^2)], \quad (5.8)$$

where

$$F_{d,2}(x) = \int_0^\infty dz \exp\left(-\frac{xz}{(2\pi)^2}\right) \left[\left(\sum_{l=-\infty}^{\infty} e^{-zl^2} \right)^d - 1 - \left(\frac{\pi}{z} \right)^{d/2} \right]. \quad (5.9)$$

Some particular values of the constant $F_{d,2}(0)$ and a method of calculation are given in [32].

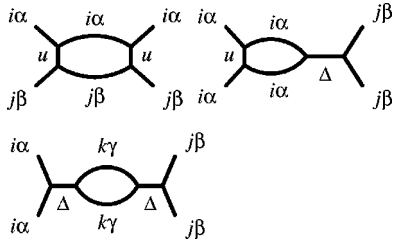
At the fixed point $\hat{u} = \hat{u}^*(n)$, $\hat{\Delta} = \hat{\Delta}^*(n)$, up to the first order in ϵ , the terms proportional to $\ln L$ cancel and Eq. (5.8) can be written in the following scaling form:

$$\tilde{t}(n)L^2 = y - \frac{3(n-1)N}{16(N-1)-nN(N+8)} [y \ln y + 4F_{d,2}(y)] \epsilon, \quad (5.10)$$

where the scaling variable $y = tL^{1/\nu(n)}$ has been introduced.

In the $n=0$ limit, the expression for the exponent measuring the divergence of the correlation length is [16,17]

$$\frac{1}{\nu_R} \equiv \frac{1}{\nu(0)} = 2 - \frac{3N}{8(N-1)} \epsilon, \quad (5.11)$$


 FIG. 2. One-loop contributions to the couplings u and Δ .

instead of the critical exponent for the pure case

$$\frac{1}{\nu_p} = 2 - \frac{N+2}{N+8} \epsilon. \quad (5.12)$$

The scaling form (5.10) can be written for this case as

$$\hat{t}(0)L^2 = y + \frac{3N}{16(N-1)} [y \ln y + 4E_{4,2}(y)] \epsilon, \quad (5.13)$$

where $y = tL^{1/\nu_R}$.

From the above expression one can obtain the large- L asymptotic form of the $T_c(\infty)$ shift, i.e.,

$$T_c(L) - T_c(\infty) \sim L^{-1/\nu_R}. \quad (5.14)$$

In Eq. (5.14), $T_c(L) := \overline{T_c(\varphi, L)}$ denotes the average pseudocritical temperature [$T_c(\varphi, L)$ is pseudocritical temperature for a specific random realization $\varphi(x)$] and $T_c(\infty) = \lim_{L \rightarrow \infty} T_c(L)$. Equation (5.14) was suggested in [2]. Combined with the phenomenological FSS theory it gave rise to the lack of SA, and is confirmed by numerical studies (see [4]). Here it is verified independently and directly.

VI. RENORMALIZATION OF THE COUPLING CONSTANTS: LOWEST ORDER IN ϵ

We perform the renormalization in a similar way also for $\tilde{u}(n)$ and $\tilde{\Delta}(n)$ by taking into account the diagrammatic contributions from u and Δ shown in Fig. 2. The result is

$$\tilde{u}(n) = uZ_u - u_L, \quad (6.1a)$$

$$\tilde{\Delta}(n) = \Delta Z_\Delta + \Delta_L, \quad (6.1b)$$

where uZ_u and ΔZ_Δ are the one-loop counterterms for the coupling constants and

$$u_L = - \left[u^2 \frac{N+8}{6} - 6u\Delta \right] \frac{1}{L^d} \sum_k' \frac{1}{(k^2+t)^2}, \quad (6.2a)$$

$$\Delta_L = - \left[u\Delta \frac{N+2}{3} - \frac{8+nN}{2} \Delta^2 \right] \frac{1}{L^d} \sum_k' \frac{1}{(k^2+t)^2} \quad (6.2b)$$

are the corresponding finite-size corrections. As one can see, the summand in Eq. (6.2) can be expressed as the first derivative of the summand of Eq. (5.7) with respect to t . So, at

the fixed point, algebraic transformations similar to those performed in the preceding section lead to

$$\tilde{u}^*(n)L^\epsilon = u^*(n) \left[1 + \frac{1}{2}(1 + \ln y) \epsilon + 2\epsilon F'_{4,2}(y) \right], \quad (6.3a)$$

$$\tilde{\Delta}^*(n)L^\epsilon = \Delta^*(n) \left[1 + \frac{1}{2}(1 + \ln y) \epsilon + 2\epsilon F'_{4,2}(y) \right], \quad (6.3b)$$

where the prime indicates that we have the derivative of the function $F_{4,2}(y)$ with respect to its argument.

The results for the disordered system simply follow by setting $n=0$. From the results for the shift of the critical temperature (5.10) and the renormalization of the coupling constant u , given by Eq. (6.3a), we reproduce the results for the pure FSS case by setting $\Delta=0$ and $n=0$. Moreover, this result still holds even if we find the FSS corrections after the analytical continuation to $n=0$, expressing the commutativity of the problem.

VII. VERIFICATION OF FSS

Let us consider the scaling variables

$$\mu(n) = \tilde{t}(n)L^{d/2/\sqrt{\tilde{u}(n)}}, \quad \lambda(n) = \tilde{\Delta}(n)/\tilde{u}(n). \quad (7.1)$$

At the fixed point they can be expressed in terms of scaling variable $y = tL^{1/\nu(n)}$,

$$\begin{aligned} \mu^*(n) &= \frac{1}{\sqrt{u^*(n)}} \\ &\times \left\{ y - \frac{1}{4}y \left[1 + \frac{(4-N)(4-nN)}{16(N-1)-nN(N+8)} \ln y \right] \epsilon \right. \\ &\quad \left. - \frac{12N(n-1)}{16(N-1)-nN(N+8)} F_{4,2}(y) \epsilon - y F'_{4,2}(y) \epsilon \right\} \end{aligned} \quad (7.2)$$

and

$$\lambda^*(u) = \frac{4-N}{3(4-nN)}. \quad (7.3)$$

In the limit $n=0$, Eqs. (7.2) and (7.3) yield the following scaling variables describing the disordered system (2.1):

$$\begin{aligned} \mu^* := \mu^*(0) &= \frac{1}{\sqrt{u^*(0)}} \left\{ y - \frac{1}{4}y \left[1 + \frac{4-N}{4(N-1)} \ln y \right] \epsilon \right. \\ &\quad \left. + \frac{3N}{4(N-1)} F_{4,2}(y) \epsilon - y F'_{4,2}(y) \epsilon \right\}, \end{aligned} \quad (7.4)$$

where $y = tL^{1/\nu_R}$, and

$$\lambda^* := \lambda^*(0) = \frac{4-N}{12}. \quad (7.5)$$

These equations verify the finite-size scaling hypotheses and show that we are really dealing with a one-variable problem,

TABLE I. Numerical values for the Binder cumulant B from Eq. (4.13) and the relative variance R from Eq. (4.14) in the mean-field regime, i.e., $d \geq 4$.

N	Random		Pure	
	B	R	B	R
1	0.216 368	0.310 240	0.270 520	0
2	0.451 486	0.111 381	0.470 401	0
3	0.533 513	0.038 365	0.543 053	0
4	0.575 587	0	0.575 587	0

since the second variable λ^* is a fixed universal number. At the critical point $t=0$, since the constant $F_{4,2}(0) = -8 \ln 2$, see Ref. [32], we have

$$\mu_0^* := \mu^*|_{t=0} = -\frac{N \ln 2}{\pi} \left(\frac{3\epsilon}{N-1} \right)^{1/2}. \quad (7.6)$$

Numerical values for different thermodynamic quantities can be obtained with the help of Eqs. (7.5) and (7.6). Note that the scaling variable μ_0^* is proportional to $\sqrt{\epsilon}$. Consequently all the ϵ -expansion results will be expressed in powers of $\sqrt{\epsilon}$, as was the case for the pure systems (see [13], for example).

VIII. CUMULANTS AND SELF-AVERAGING

In Ref. [6], the Binder cumulant B and the relative variance R [Eqs. (4.13) and (4.14)] have been calculated analytically and numerically at the critical point $T=T_c$ in the asymptotic regime $\lambda \cong \frac{1}{4}$ for $N=1$ and $d=4$. In Ref. [33] (see also [4]), the same quantities were calculated numerically for $N=1$ and $d=3$. In both cases the results show that the system exhibits a lack of SA.

In the remainder of this section we concentrate on the calculation of the cumulants B and R [Eqs. (4.13) and (4.14)] in cases $d \geq 4$ and $d=4-\epsilon$. The reason for considering the case $d \geq 4$ is in its simple analytical nonperturbative treatment. Although the results based on the ϵ expansion give only a qualitative description of the three-dimensional physics, we hope that they shed some light at least on the applicability of the theory for studying diluted models.

Let us first note that if $1 < N < 4$ and $d=4-\epsilon$, the case under consideration applies to the situation [see Eq. (5.11)] where $\nu_R > 2/d$ and randomness is relevant (R case). Up to the first order in ϵ , due to the RG arguments, no SA must be expected near the critical point [2]. This statement is supported also by our RG calculations. In Tables I and II we present the corresponding universal numbers for B and R at $d \geq 4$ and $d=3$ in the region $Lt^{\nu_R} = L/\xi \ll 1$, i.e., in the vicinity of the critical point. The calculations are performed with variable $\mu=0$ for $d \geq 4$ and $\mu = \mu_0^*$ from Eq. (7.6) (setting $\epsilon=1$) for $d=3$, and with variable λ taken from Eq. (7.5) in both cases. The asymptotic behavior for small μ is presented in the Appendix. The numerical values of B and R in the random case and for $N=1$, presented in Table I are in full agreement with those obtained in Ref. [6], while those of B

TABLE II. Numerical values for the Binder cumulant B from Eq. (4.13) and the relative variance R from Eq. (4.14) at $d=3$.

N	Random		Pure	
	B	R	B	R
1			0.400 024	0
1.001	0.666 334	0.000 427	0.400 328	0
2	0.602 793	0.061 279	0.547 496	0
3	0.625 783	0.022 688	0.592 813	0
4	0.640 628	0	0.614 002	0

for the pure case and $N=1$ (Tables I and II) are in full agreement with Ref. [13].

The random case $N=1$ for $d < 4$ cannot be considered within the present expansion, because of the apparent divergence of μ_0^* that takes place to the used order in ϵ . Up to now there are only numerical values $B = \frac{2}{3}$, $g_2 = 0.448$, and $R = g_2 = 0.510(7)$ obtained in [33] through Monte Carlo simulations. What is possible to calculate here are the corresponding values of B and R very close to $N=1$, e.g., for $N=1.001$. For completeness these results are presented in Table II. More generally, one can see that if $N \rightarrow 1$, then $B \rightarrow \frac{2}{3}$ and $R \rightarrow 0$, i.e., the system exhibits SA. This evident discrepancy with the reality is due to the wrong assumption that some information about the random case $N=1$ can be obtained from the above formulas in this limiting case. As it was pointed out, the correct treatment of the case $N=1$ seems to be a more difficult computational problem.

The finite-size correction to the bulk critical behavior of the cumulants B and R in the region $Lt^{\nu_R} = L/\xi \gg 1$, i.e., away from the critical point, are obtained with the help of the asymptotics $\mu \gg 1$, given in the Appendix [Eqs. (A7) and (A8)]. According to the analysis presented there, we obtain for Binder's cumulant.

$$B = 1 - \frac{1}{3} \left(1 + \frac{2}{N} \right) \left[1 + \frac{3\lambda - 1}{3\mu^2} \right] + O\left(\frac{1}{\mu^3} \right). \quad (8.1)$$

For the cumulant R we get

$$R = \frac{\lambda}{\mu^2} + O\left(\frac{1}{\mu^3} \right). \quad (8.2)$$

The final results can be obtained by replacing λ and μ by their respective expressions evaluated at the fixed points of the model given in Eqs. (7.4) and (7.5). So, to the lowest order in ϵ , we have

$$B = 1 - \frac{1}{3} \left(1 + \frac{2}{N} \right) + O\left(\frac{\xi}{L} \right) \quad (8.3)$$

and

$$R = \frac{4-N}{8(N-1)} \epsilon \left(\frac{\xi}{L} \right)^4 + O\left(\frac{\xi^5}{L^5} \right). \quad (8.4)$$

It is interesting to compare Eq. (8.1) with the corresponding result for the pure system [34],

$$B_{\text{pure}} = 1 - \frac{1}{3} \left(1 + \frac{2}{N} \right) \left[1 - \frac{1}{3\mu_p^2} + O\left(\frac{1}{\mu_p^4}\right) \right]. \quad (8.5)$$

Up to the lowest order in $1/\mu$ they coincide for $1 < N < 4$; moreover, for $N = 4 - \delta$ (with $\delta \ll 1$), we have $\mu = \mu_p + O(\epsilon\delta)$ and $B = B_{\text{pure}} + O(\epsilon\delta)$. The result (8.4) confirms the statement [2] that away from the critical point, a strong SA emerges in the system as $R \rightarrow 0$ with $L/\xi \gg 1$.

IX. CONCLUSIONS

In the present paper we propose a general scheme for the FSS analysis of finite-size disordered $O(N)$ system. The method we use here is an extension of the field-theoretical methods used to analyze FSS properties in pure systems. The nature of the symmetry (obtained as a consequence of the use of the replica trick, which removes the disorder) of the model complicates the perturbative structure of the theory in comparison with the corresponding $O(N)$ pure one. Recall that, the final results for the disordered system are obtained by making the number of replicas vanishing. Our results concern mainly systems with number of components larger than 1, i.e., non-Ising systems. Their extension to Ising systems requires higher loop calculations because of the degeneracy of the one-loop order RG equations.

Our main results are related to the formulation of the problem for some number of components N of the fluctuating field for dimensions $d > 4$ and $d = 4 - \epsilon$. Due to the presence of randomness, it is shown that we are dealing with a two-variable problem with scaling variables $\mu = tL^{d/2}u^{-1/2}$ and $\lambda = \Delta/u$. In the mean-field regime $d > 4$ our results are a generalization for $N > 1$ of those obtained in [6] for $N = 1$. Evaluating numerically the corresponding analytic expressions for the Binder's cumulant B and the relative variance R , we demonstrate a monotonic increase of B as a function of N in both pure and random cases and a monotonic decrease of R (to zero for $N = 4$) in the random case (see Table I).

The ϵ expansion to first order in ϵ shows that close to the critical temperature, one can express the physical observables, the shift of the critical temperature, and the renormalization of the coupling constants in terms of μ and λ . In the random fixed point the parameter λ takes a universal value [see Eq. (7.5)]. It is found that the distance, over which the bulk critical temperature is shifted is proportional to L^{-1/ν_R} in agreement with the statement of Ref. [2]. This result, combined with phenomenological FSS, gives rise to the lack of SA. The scaling parameters μ and λ also enter in the final expression for the Binder cumulant (4.13) and the relative variances (4.14), giving explicit expressions for them in the different asymptotic regimes. The numerical calculation of the above parameters permits also the verification of the FSS and SA (see Table II). The latter is shown to be absent in the regime $\nu_R > 2/d$, where randomness is relevant, and our analysis explicitly shows that in this regime the relative variance is always nonzero for the case of stability of the RG solutions, i.e., $1 < N < 4$ and $d = 4 - \epsilon$.

One can also try to repeat the analysis without the use of the replica trick. For this, one needs to define in a proper way the procedure in the zero-mode approximation, when rescal-

ing the random part of the Hamiltonian for a finite system. One can realize that this is the $\mathbf{k} = 0$ part of this term, which will give a contribution to the free energy. This scheme permits formally to end with the same expressions for the shift of the critical temperature and the renormalization constants, as we did within the replica formalism.

In our opinion, the present FSS study can be also applied in the ‘‘canonical’’ case [5], where the disorder is characterized by a constant total number of the occupied sites (or bonds), instead of the constant average density. We hope that results, similar to the bulk case, will also hold in the case of finite geometry, relating in this way our theoretical findings with the Monte Carlo simulations.

ACKNOWLEDGMENTS

The authors thank J. Ruiz-Lorenzo for useful comments. They also acknowledge the hospitality of the Abdus Salam International Center for Theoretical Physics, Trieste, where part of this work was done. E.K. and N.S.T. acknowledge the financial support of Associate & Federation Schemes of ICTP. E.K. is also supported by the Spanish DGES Contract No. PB97-0076.

APPENDIX: FINITE-SIZE SCALING BEHAVIOR OF THE EVEN MOMENTS OF THE ORDER PARAMETER

In this appendix we present the mathematical details of how to obtain the asymptotes of the averages

$$\overline{\mathcal{M}_{2m}} = \frac{(uL^d)^{-m/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \frac{\mathcal{I}_{N+2m}(\mu + \sqrt{\lambda}x)}{\mathcal{I}_N(\mu + \sqrt{\lambda}x)} e^{-x^2/2}, \quad (A1a)$$

$$\overline{(\mathcal{M}_2)^2} = \frac{(uL^d)^{-1}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \left[\frac{\mathcal{I}_{N+2}(\mu + \sqrt{\lambda}x)}{\mathcal{I}_N(\mu + \sqrt{\lambda}x)} \right]^2 e^{-x^2/2}, \quad (A1b)$$

where we have introduced the function

$$\mathcal{I}_N(z) = S_N \int_0^{\infty} d\Phi \Phi^{(N-1)} \exp\left\{-\frac{1}{2}[z\Phi^2 + \frac{1}{12}\Phi^4]\right\}. \quad (A2)$$

The integral in the definition of function $\mathcal{I}_N(z)$ given by Eq. (A2) may be evaluated in terms of parabolic cylinder functions $D_\nu(z)$ using the identity [35]

$$\int_0^{\infty} x^{\nu-1} e^{-\beta x^2 - \gamma x} dx = (2\beta)^{-\nu/2} \Gamma(\nu) \exp\left(\frac{\gamma^2}{8\beta}\right) D_{-\nu}\left(\frac{\gamma}{\sqrt{2\beta}}\right). \quad (A3)$$

The result is

$$\mathcal{I}_N(z) = (12\pi^2)^{N/4} \exp\left(\frac{3z^2}{4}\right) D_{-N/2}(\sqrt{3}z). \quad (A4)$$

Now the above integrand in Eq. (A1a) can be rewritten in a very simple form

$$\begin{aligned}\mathcal{M}_{2m}(x) &:= \frac{\mathcal{I}_{N+2m}(\mu + \sqrt{\lambda}x)}{\mathcal{I}_N(\mu + \sqrt{\lambda}x)} \\ &= (12\pi^2)^{m/2} \frac{D_{-m-N/2}[(\mu + \sqrt{\lambda}x)\sqrt{3}]}{D_{-N/2}[(\mu + \sqrt{\lambda}x)\sqrt{3}]}.\end{aligned}\quad (\text{A5})$$

For small $\mu \ll 1$ (i.e., in the vicinity of the critical point) the asymptotic form of the ratio (A5) is given by

$$\begin{aligned}\mathcal{M}_{2m}(x) &= (12\pi^2)^{m/2} \left\{ \frac{D_{-m-N/2}(x\sqrt{3\lambda})}{D_{-N/2}(x\sqrt{3\lambda})} \right. \\ &+ \frac{\mu\sqrt{3}}{2} \left[N \frac{D_{-N/2-1}(x\sqrt{3\lambda})D_{-m-N/2}(x\sqrt{3\lambda})}{[D_{-N/2}(x\sqrt{3\lambda})]^2} \right. \\ &\left. \left. - (2m+N) \frac{D_{-m-N/2-1}(x\sqrt{3\lambda})}{D_{-N/2}(x\sqrt{3\lambda})} \right] + O(\mu)^2 \right\}.\end{aligned}\quad (\text{A6})$$

In the mean-field regime and at the critical point we have $\mu = 0$, and \mathcal{M}_{2m} is equal to the first term in the right-hand side of Eq. (A6).

For large $\mu \gg 1$, the asymptotic behavior of the ratio \mathcal{M}_{2m} is obtained with the help of the well-known Watson's Lemma.

Lemma. (See, for example, [36]) Suppose $\alpha > 0$, $\beta > 0$ and $f(x)$ is an analytic function in a neighborhood of $x = 0$,

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k, \quad |x| < R,$$

and that

$$|f(x)| \leq c_1 e^{c_2 x^\alpha}, \quad x \in [R, X]$$

for positive constants c_1, c_2 . Then

$$\int_0^X x^{\beta-1} e^{-sx^\alpha} f(x) dx \sim \frac{1}{\alpha} \sum_{k=0}^{\infty} s^{-(k+\beta)/\alpha} \Gamma\left(\frac{k+\beta}{\alpha}\right) \frac{f^{(k)}(0)}{k!},$$

as $s \rightarrow \infty$ in the sector $|\arg s| < \pi/2$.

According to this Lemma, from Eqs. (A2) (with $z = \mu + x\sqrt{\lambda}$) and (A5), we have

$$\mathcal{M}_2(x) = (12\pi^2)^{1/2} \frac{N}{\mu} \left[1 - \frac{x\sqrt{\lambda}}{\mu} + \frac{6x^2\lambda - N - 2}{6\mu^2} + O\left(\frac{1}{\mu^3}\right) \right] \quad (\text{A7})$$

and

$$\begin{aligned}\mathcal{M}_4(x) &= 12\pi^2 \frac{N(N+2)}{\mu^2} \left[1 - \frac{2x\sqrt{\lambda}}{\mu} + \frac{9x^2\lambda - N - 3}{3\mu^2} \right. \\ &\left. + O\left(\frac{1}{\mu^3}\right) \right].\end{aligned}\quad (\text{A8})$$

Using the asymptotic of \mathcal{M}_2 and \mathcal{M}_4 for large μ , we can get the behavior of the cumulants R and B in both cases $d \geq 4$ and $d = 4 - \epsilon$. They are given by

$$B = 1 - \frac{1}{3} \left(1 + \frac{2}{N} \right) \left[1 + \frac{3\lambda - 1}{3\mu^2} \right] + O\left(\frac{1}{\mu^3}\right). \quad (\text{A9})$$

For the cumulant R we get

$$R = \frac{\lambda}{\mu^2} + O\left(\frac{1}{\mu^3}\right). \quad (\text{A10})$$

-
- [1] S. Wiseman and E. Domany, Phys. Rev. E **52**, 3469 (1995).
[2] A. Aharony and A. B. Harris, Phys. Rev. Lett. **77**, 3700 (1996).
[3] F. Pázmándi, R. T. Scalettar, and G. T. Zimányi, Phys. Rev. Lett. **79**, 5130 (1997).
[4] S. Wiseman and E. Domany, Phys. Rev. E **58**, 2938 (1998).
[5] A. Aharony, A. B. Harris, and S. Wiseman, Phys. Rev. Lett. **81**, 252 (1998).
[6] H. G. Ballesteros, L. A. Fernández, V. Martín-Mayor, A. M. Sudupe, G. Parisi, and J. J. Ruiz-Lorenzo, Nucl. Phys. B **512** [FS], 681 (1998).
[7] M. I. Marques and J. A. Gonzalo, Phys. Rev. E **60**, 2394 (1999).
[8] K. Bernardet, F. Pázmándi, and G. C. Batrouni, Phys. Rev. Lett. **84**, 4477 (2000).
[9] M. Barber, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic, London, 1983), Vol. 8.
[10] *Finite-size Scaling and Numerical Simulations of Statistical Systems*, edited by V. Privman (World Scientific, Singapore, 1990).
[11] J. G. Brankov, D. M. Danchev, and N. S. Tonchev, *Theory of Critical Phenomena in Finite-Size Systems; Scaling and Quantum Effects* (World Scientific, Singapore, 2000).
[12] K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986).
[13] E. Brézin and J. Zinn-Justin, Nucl. Phys. B **257** [FS14], 867 (1985).
[14] J. Rudnick, H. Guo, and D. Jasnow, J. Stat. Phys. **41**, 751 (1985).
[15] D. E. Khemel'nitskii, Zh. Eksp. Theor. Fiz. **68**, 1960 (1975) [Sov. Phys. JETP **41**, 981 (1975)].
[16] T. Lubensky, Phys. Rev. B **11**, 3575 (1975).
[17] G. Grinstein and A. Luther, Phys. Rev. B **13**, 1329 (1976).
[18] C. Jayaprakash and H. J. Katz, Phys. Rev. B **16**, 3987 (1977).
[19] B. N. Shalaev, Zh. Eksp. Theor. Fiz. **73**, 2301 (1977) [Sov. Phys. JETP **46**, 1244 (1977)].
[20] G. Jug, Phys. Rev. B **27**, 609 (1983).
[21] E. Korutcheva and D. Uzunov, Phys. Lett. A **106**, 175 (1984).
[22] M. Mézard, G. Parisi, and M.-A. Virasoro, *Spin Glass Theory and Beyond* (World Scientific, Singapore, 1987).
[23] A. Pelissetto and E. Vicari, Phys. Rev. B **62**, 6393 (2000).

- [24] The technique that involves application of the RG transformations to the disordered system directly [16] gives the same fixed points as the technique that involves first removing the disorder by averaging and then employing the RG transformation [17]. For a direct nonperturbative derivation of this equivalence see V. J. Emery, Phys. Rev. B **11**, 239 (1975).
- [25] The indices “ r ” and “ p ” will refer to the “random” and the effective “pure” system.
- [26] E. Brezin, J. Phys. (Paris) **43**, 15 (1982).
- [27] A. J. Bray, T. McCarthy, M. A. Moore, J. D. Reger, and A. P. Young, Phys. Rev. B **36**, 2212 (1987).
- [28] A. J. McKane, Phys. Rev. B **49**, 12 003 (1994).
- [29] G. Álvarez, V. Martin-Mayor, and J. J. Ruiz-Lorenzo, J. Phys. A **33**, 841 (2000).
- [30] E. Brézin, J. C. Le Guillou, and J. Zinn-Justin, Phys. Rev. B **10**, 892 (1974).
- [31] K. De’Bell and D. J. W. Geldart, Phys. Rev. B **32**, 4763 (1985).
- [32] H. Chamati and N. S. Tonchev, J. Phys. A **33**, L167 (2000).
- [33] H. G. Ballesteros, L. A. Fernández, V. Martin-Mayor, A. M. Sudupe, G. Parisi, and J. J. Ruiz-Lorenzo, Phys. Rev. B **58**, 2740 (1998).
- [34] H. Chamati and D. M. Danchev (unpublished).
- [35] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1965).
- [36] M. V. Fedoruk, *Asymptotics: Integrals and Series* (Nauka, Moscow, 1987).